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Primordial Non-Gaussianity of Gravitational Waves in General Covariant Hořava-Lifshitz Gravity

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In this paper, we study 3-point correlation function of primordial gravitational waves generated in the de Sitter background in the framework of the general covariant Hořava-Lifshitz gravity with an arbitrary coupling constant λ. We find that, to the leading order, the interaction Hamiltonian receives contributions from four terms built of the 3-dimensional Ricci tensor R_{ij} of the leaves $t =$ constant. In particular, the 3D Ricci scalar R yields the same k-dependence as that in general relativity, but with different magnitude due to coupling with the $U(1)$ field A and a UV history. The two terms $R_{ij}R^{ij}$ and $\left(\nabla^i R^{jk}\right)\left(\nabla_i R_{jk}\right)$ exhibit a peak at the squeezed limit. The term $R^i_j R^j_k R^k_i$ favors the equilateral shape when spins of the three tensor fields are the same but peaks in between the equilateral and squeezed limits when spins are mixed. This is due to the effects of the polarization tensors. Hence, a detection of the squeezed shape or absence of in the bispectrum cannot rule out completely higher derivative gravity theories, at least for R^2 theories. The consistency with the recently-released Planck observations on non-Gaussianity is also discussed.

I. INTRODUCTION

The Hořava-Lifshitz (HL) theory of quantum gravity, proposed recently by Hořava [1], motivated by the Lifshitz scalar field theory in solid state physics [2], has attracted a great deal of attention, due to its several remarkable features in cosmology as well as some challenging questions, such as instability, ghost, strong coupling, and different speeds in the gravitational sector [3]. To resolve these issues, various models have been proposed, along two different lines, one with the projectability condition [4–8],

$$N = N(t), \tag{1.1}$$

and the other without it [9–12], where N denotes the lapse function in the Arnowitt, Deser and Misner decompositions [13]. In particular, Hořava and Melby-Thompson (HMT) proposed to enlarge the foliation-preserving diffeomorphisms of the original model,

$$\delta t = -f(t), \quad \delta x^i = -\zeta^i(t, \mathbf{x}), \tag{1.2}$$

often denoted by Diff($M,\ \mathcal{F}$), to include a local U(1) symmetry, so that the reformulated theory has the symmetry [5],

$$U(1) \ltimes \text{Diff}(M,\ \mathcal{F}). \tag{1.3}$$

With such an enlarged symmetry, the spin-0 gravitons, which appear in the original model of the HL theory [1], are eliminated [5, 6], and as a result, all the problems related to them, including instability, ghost, and strong coupling, are resolved automatically. This was initially done with $\lambda = 1$, where λ characterizes the deviation of the theory from general relativity (GR) in the infrared, as one can see from Eqs.(2.1) and (2.2) given below. It was soon generalized to the case with any λ [7], in which it was shown that the spin-0 gravitons are also eliminated [7, 8], so that the above mentioned problems are resolved in the gravitational sector even with any λ. In the matter sector, the strong coupling problem, first noted in [8], can be solved by introducing a mass M_* so that $M_* < \Lambda_\omega$, where M_* denotes the suppression energy of high order operators, and Λ_ω the would-be energy scale, above which matter becomes strongly coupled [14], similar to the non-projectability case without the enlarged symmetry [15]. The consistence of this model with solar system tests was investigated recently [16], and found that it is consistent with observations, provided that the gauge field and Newtonian prepotential are part of the metric, so that the line element ds^2 is invariant not only under the coordinate transformations (1.2), but also under the local U(1) gauge transformations,

$$\begin{aligned} \delta_\alpha N &= 0, \quad \delta_\alpha N_i = N\nabla_i\alpha, \quad \delta_\alpha g_{ij} = 0, \\ \delta_\alpha A &= \dot{\alpha} - N^i\nabla_i\alpha, \quad \delta_\alpha\varphi = -\alpha, \end{aligned} \tag{1.4}$$

where α denotes the U(1) generator, and $\dot{\alpha} \equiv \partial\alpha/\partial t$, and N^i and g_{ij} are, respectively, the shift vector and the 3-metric of the leaves $t =$ constant, with $N_i \equiv g_{ij}N^j$. A denotes the U(1) gauge field, and φ the Newtonian pre-potential. For detail, see [16].

In this paper, we shall work within the HMT framework of the HL theory with the projectability condition [5–8], although our main results are expected to hold equally in the non-projectable case [10], as the tensor perturbations are almost the same in both cases [8, 10, 11]. For the current status of the models proposed in [4, 5, 7, 9, 10], we refer readers to [16, 17].

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Continuing our previous study of the statistics of the primordial perturbations in single-field slow-roll inflation in the HMT framework [18, 19], we study here the 3-point function of the tensor perturbations in de Sitter background. Non-Gaussianity of tensor perturbations have been studied intensively recently in various theories of gravity. But, as far as we know, this is the first time to investigate this problem within the framework of the HL theory. In particular, [20] studied the non-Gaussian characteristics in the most general single-field inflation model with second-order field equations and found that the interaction at cubic order is composed of only two terms, which generate squeezed and equilateral shapes. On the other hand, [21] focused on possible parity violations. In this paper we shall focus on the shapes of the bispectrum generated by the various terms in the gravity sector, especially the higher spatial derivative terms and leave the topic of parity violations to future studies. For detail on the power spectrum and its scale dependence of the tensor perturbations, we refer readers to section 6.2 of [18].

The rest of the paper is organized as follows. In Section II we give a brief review of the non-relativistic general covariant Hořava-Lifshitz theory of gravity with the projectability condition and an arbitrary coupling constant λ. The interaction Hamiltonian $H_{\rm I}$ is analyzed in Section III, and was found to receive contributions from only four terms in the potential part of the theory. Section IV performs the integration of the mode functions of the quantized fields, where we also present conditions with which the UV history in the integral can be ignored as a subleasing error term. We then plot various shapes of the bispectrum generated by the four terms in Section V. In Section VI, we consider the non-Gaussianity constraints from Planck observations released recently to obtain restrictions on the energy scale M_* and the inflation energy H. Finally, in Section VII we summarize our main results.

II. GENERAL COVARIANT HL GRAVITY WITH PROJECTABILITY CONDITION

The action of the general covariant HL theory of gravity with the projectability condition can be written as [5–8],

$$
\begin{aligned}
S &= \frac{1}{16\pi G}\int dtd^3xN\sqrt{g}\Big(\mathcal{L}_K-\mathcal{L}_V+\mathcal{L}_\varphi+\mathcal{L}_A+\mathcal{L}_\lambda\Big)\\
&\quad+\int dtd^3xN\sqrt{g}\mathcal{L}_M, \qquad (2.1)
\end{aligned}
$$

where $g=\det(g_{ij})$, G is the Newtonian constant, and

$$
\begin{aligned}
\mathcal{L}_K &= K_{ij}K^{ij}-\lambda K^2,\\
\mathcal{L}_V &= \zeta^2 g_0+g_1R+\frac{1}{\zeta^2}\left(g_2R^2+g_3R_{ij}R^{ij}\right)\\
&\quad+\frac{1}{\zeta^4}\Big(g_4R^3+g_5R\;R_{ij}R^{ij}+g_6R^i_jR^j_kR^k_i\Big)\\
&\quad+\frac{1}{\zeta^4}\left[g_7R\Delta R+g_8\left(\nabla_iR_{jk}\right)\left(\nabla^iR^{jk}\right)\right],\\
\mathcal{L}_\varphi &= \varphi\mathcal{G}^{ij}\Big(2K_{ij}+\nabla_i\nabla_j\varphi\Big), \qquad (2.2)\\
\mathcal{L}_A &= \frac{A}{N}\Big(2\Lambda_g-R\Big),\\
\mathcal{L}_\lambda &= \left(1-\lambda\right)\Big[\left(\Delta\varphi\right)^2+2K\Delta\varphi\Big]. \qquad (2.3)
\end{aligned}
$$

Here λ characterizes the deviation of the theory from general relativity (GR) in the infrared, as mentioned previously, $\Delta\equiv g^{ij}\nabla_i\nabla_j$, Λ_g is a coupling constant, the Ricci and Riemann tensors R_{ij} and R^i_{jkl} all refer to the 3-metric g_{ij}, and

$$
\begin{aligned}
K_{ij} &= \frac{1}{2N}\left(-\dot{g}_{ij}+\nabla_iN_j+\nabla_jN_i\right),\\
\mathcal{G}_{ij} &= R_{ij}-\frac{1}{2}g_{ij}R+\Lambda_g g_{ij}. \qquad (2.4)
\end{aligned}
$$

The coupling constants $g_s\,(s=0,1,2,\dots 8)$ are all dimensionless, and

$$
\Lambda=\frac{g_0}{32\pi G}, \qquad (2.5)
$$

is the cosmological constant. The relativistic limit in the IR, on the other hand, requires,

$$
g_1=-1. \qquad (2.6)
$$

$\mathcal{L}_M$ is the Lagrangian of matter fields, and for a scalar field χ, it is given by [18, 22],

$$
\begin{aligned}
\mathcal{L}_M &= \mathcal{L}^{(0)}_\chi+\mathcal{L}^{(A,\varphi)}_\chi,\\
\mathcal{L}^{(0)}_\chi &= \frac{f(\lambda)}{2N^2}\Big(\dot\chi-N^i\nabla_i\chi\Big)^2-\mathcal{V},\\
\mathcal{V} &= V\left(\chi\right)+\left(\frac{1}{2}+V_1\left(\chi\right)\right)\left(\nabla\chi\right)^2+V_2\left(\chi\right)\mathcal{P}_1^2\\
&\quad+V_3\left(\chi\right)\mathcal{P}_1^3+V_4\left(\chi\right)\mathcal{P}_2+V_5\left(\chi\right)\left(\nabla\chi\right)^2\mathcal{P}_2\\
&\quad+V_6(\chi)\mathcal{P}_1\mathcal{P}_2, \qquad (2.7)\\
\mathcal{L}^{(A,\varphi)}_\chi &= \frac{A-\mathcal{A}}{N}\Big[c_1\left(\chi\right)\Delta\chi+c_2\left(\chi\right)\left(\nabla\chi\right)^2\Big]\\
&\quad-\frac{f}{N}\Big(\dot\chi-N^i\nabla_i\chi\Big)\Big(\nabla^k\varphi\Big)\Big(\nabla_k\chi\Big)\\
&\quad+\frac{f}{2}\Big[\Big(\nabla^k\varphi\Big)\Big(\nabla_k\chi\Big)\Big]^2, \qquad (2.8)
\end{aligned}
$$

with $c_1\left(\chi\right),c_2\left(\chi\right),V(\chi)$ and $V_n(\chi)$ being arbitrary functions of χ, and

$$
\mathcal{P}_n\equiv\Delta^n\chi. \qquad (2.9)
$$

For detail, we refer readers to [8, 18]. In the following we shall use directly the symbols and conversions from these papers, without further explanations.

III. THE INTERACTION HAMILTONIAN

We assume that no tensor perturbations exist in the matter sector, and the tensor perturbations of the metric around a spatially flat FLRW metric is [20]

$$N = a, \quad N^i = 0, \quad g_{ij} = a^2 \left(e^h\right)_{ij}, \tag{3.1}$$

where

$$\left(e^h\right)_{ij} = \delta_{ij} + h_{ij} + \frac{1}{2} h_{ik} h^k_j + \frac{1}{6} h_{ij} h^{jk} h_{ki} + \cdots . \tag{3.2}$$

The metric perturbation is defined in this way such that there is no cubic term involving two time derivatives in GR [23]. The small dimensionless quantity h_{ij} satisfies the transverse and traceless condition $\partial^i h_{ij} = 0 = h^i_i$. Moreover, its indices are lowered and raised by δ_{ij} and δ^{ij}. Thus for simplicity of notations, in this paper we shall not distinguish super-indices and sub-indices, with the understanding that when an index appears twice, summation over that index is performed. We further introduce a short hand notation

$$\left(h^n\right)_{ij} \equiv h_{ik_1} h_{k_1 k_2} \cdots h_{k_{n-1} j}. \tag{3.3}$$

With these perturbations, the interaction Hamiltonian at leading order is found to receive contributions from only four terms

$$R, \quad R_{ij} R^{ij}, \quad R^i_j R^j_k R^k_i, \quad \left(\nabla^i R^{jk}\right)\left(\nabla_i R_{jk}\right). \tag{3.4}$$

The kinetic part of the action S_K, like the case in GR, does not contribute to H_I even though λ now could differ from 1 [cf. Eq.(2.2)]. We find that,

$$R = \frac{a^{-2}}{4} h^{ij,ab} \left(2 h_{ia} h_{jb} - h_{ij} h_{ab}\right), \tag{3.5}$$

$$R_{ij} R^{ij} = a^{-4} \left(\partial^2 h^{ij}\right) \left\{ \left[\frac{1}{2} h_{ia,b} h_{jb,a}\right] + h^{ab} \Big[h_{ia,jb} - \frac{1}{4} h_{ab,ij} - \frac{1}{2} h_{ij,ab}\Big] \right\}, \tag{3.6}$$

$$R^i_j R^j_k R^k_i = -\frac{a^{-6}}{8} \left[\partial^2\left(h_{ij}\right)\right] \left[\partial^2\left(h_{jk}\right)\right] \left[\partial^2\left(h_{ki}\right)\right], \tag{3.7}$$

$$\begin{aligned} (\nabla^i R^{jk})(\nabla_i R_{jk}) = {}& \frac{a^{-6}}{4} \left(\partial^2 h_{ij}\right) \Big[h^{ab} \left(\partial^2 h_{ij}\right)_{,ab} - 4 h^{ai,b} \left(\partial^2 h_{bj}\right)_{,a}\Big] \\ & - \frac{a^{-6}}{4} \left(\partial^4 h_{ij}\right) \left\{ 2 \Big[h_{ia,b} h_{jb,a}\Big] - h^{ab} \Big[-4 h_{ia,bj} + 2 h_{ij,ab} + h_{ab,ij}\Big] \right\}. \end{aligned} \tag{3.8}$$

The contribution from R is the same as that obtained in [20]. The contribution from $R^i_j R^j_k R^k_i$ differs from theirs in an essential way. The model considered in [20] describes the class of single-field inflation models where the coupling between the inflaton and gravity could be different from the canonical form while Lorentz symmetry is kept. This explains why their interaction term posses time-derivatives. On the other hand, coupling of the gravitational sector with scalar matter in our model is in the canonical form [cf. Eq. (2.7)]. The higher spatial derivative terms exist because of the Lorentz symmetry breaking.

We define the Fourier image of h_{ij} here in the canonical form,

$$h_{ij} = \int \frac{d^3 k}{(2\pi)^3} \sum_{s=+,-} \varepsilon^s_{ij}(\mathbf{k}) h^s_{\mathbf{k}}(t) e^{i\mathbf{kx}}, \tag{3.9}$$

where $h^s_{\mathbf{k}}(t)$ is now a scalar quantity, and the rank-2 polarization tensors satisfy $\varepsilon^s_{ii}(\mathbf{k}) = k^i \varepsilon^s_{ij}(\mathbf{k}) = 0$ and $\varepsilon^s_{ij}(\mathbf{k}) \varepsilon^{s'}_{ij}(\mathbf{k}) = \delta_{ss'}$.[1] By a proper choice of phase, they satisfy the relation [20]

$$\left[\varepsilon^s_{ij}(\mathbf{k})\right]^* = \varepsilon^{-s}_{ij}(\mathbf{k}) = \varepsilon^s_{ij}(-\mathbf{k}). \tag{3.11}$$

Then, in the momentum space we find

$$a^2 R \rightsquigarrow -\frac{1}{6} \left\{ \Big[C_1\Big]_{\mathbf{q}}(123) - \frac{1}{4} \Big[C_2\Big]_{\mathbf{q}}(123) - \frac{1}{4} \Big[C_3\Big]_{\mathbf{q}}(123) \right\} + \text{cyclic}, \tag{3.12}$$

$$a^4 R^{ij} R_{ij} \rightsquigarrow \frac{q_1^2 + q_2^2 + q_3^2}{6} \left\{ \Big[C_1\Big]_{\mathbf{q}}(123) - \frac{1}{4} \Big[C_2\Big]_{\mathbf{q}}(123) - \frac{1}{4} \Big[C_3\Big]_{\mathbf{q}}(123) \right\} + \text{cyclic}, \tag{3.13}$$

$$a^6 R^i_j R^j_k R^k_i \rightsquigarrow \frac{\left(q_1 q_2 q_3\right)^2}{24} \Big[C_5\Big]_{\mathbf{q}}(123) + \text{cyclic}, \tag{3.14}$$

$$\begin{aligned} a^6 \left(\nabla^i R^{jk}\right)\left(\nabla_i R_{jk}\right) \rightsquigarrow {}& \frac{q_1^4 + q_2^4 + q_3^4}{6} \left\{ \Big[C_1\Big]_{\mathbf{q}}(123) - \frac{1}{4} \Big[C_2\Big]_{\mathbf{q}}(123) - \frac{1}{4} \Big[C_3\Big]_{\mathbf{q}}(123) \right\} \\ & + \frac{q_1^2 q_3^2}{6} \left\{ \Big[C_1\Big]_{\mathbf{q}}(123) - \frac{1}{4} \Big[C_3\Big]_{\mathbf{q}}(123) \right\} \\ & + \frac{q_1^2 q_2^2}{6} \left\{ \Big[C_1\Big]_{\mathbf{q}}(123) - \frac{1}{4} \Big[C_2\Big]_{\mathbf{q}}(123) \right\} \\ & + \text{cyclic}, \end{aligned} \tag{3.15}$$

[1] This set of polarization tensors is related to those introduced in [18] through

$$\varepsilon^{\pm}_{ij} = \frac{1}{2} \left(\epsilon^+_{ij} \pm i \epsilon^{\times}_{ij}\right). \tag{3.10}$$

where "cyclic" refers to the cyclic rotation of (1, 2, 3), and we've defined the symbol $\rightsquigarrow$ to have meaning of

$$= \int \prod_{j=1}^{3} \left[\frac{d^3 q_j e^{i\mathbf{x}\cdot\mathbf{q}_j}}{(2\pi)^3} \sum_{s_j=+,-} h^{s_j}_{\mathbf{q}_j}(t') \right], \tag{3.16}$$

and introduced the shorthand notations

$$\begin{aligned}
\Big[C_1\Big]_{\mathbf{q}}(123) &\equiv k_{1a}k_{1b}\varepsilon^{s_1}_{ij}(\mathbf{q}_1)\,\varepsilon^{s_2}_{ia}(\mathbf{q}_2)\,\varepsilon^{s_3}_{jb}(\mathbf{q}_3),\\
\Big[C_2\Big]_{\mathbf{q}}(123) &\equiv k_{1a}k_{1b}\varepsilon^{s_1}_{ij}(\mathbf{q}_1)\,\varepsilon^{s_2}_{ab}(\mathbf{q}_2)\,\varepsilon^{s_3}_{ij}(\mathbf{q}_3),\\
\Big[C_3\Big]_{\mathbf{q}}(123) &\equiv k_{1a}k_{1b}\varepsilon^{s_1}_{ij}(\mathbf{q}_1)\,\varepsilon^{s_2}_{ij}(\mathbf{q}_2)\,\varepsilon^{s_3}_{ab}(\mathbf{q}_3),\\
\Big[C_4\Big]_{\mathbf{q}}(123) &\equiv k_{1a}k_{1b}\varepsilon^{s_1}_{ij}(\mathbf{q}_1)\,\varepsilon^{s_2}_{ab}(\mathbf{q}_2)\,\varepsilon^{s_3}_{ab}(\mathbf{q}_3),\\
\Big[C_5\Big]_{\mathbf{q}}(123) &\equiv \varepsilon^{s_1}_{ij}(\mathbf{q}_1)\,\varepsilon^{s_2}_{jk}(\mathbf{q}_2)\,\varepsilon^{s_3}_{ki}(\mathbf{q}_3).
\end{aligned} \tag{3.17}$$

Hence, the Hamiltonian reads

$$\begin{aligned}
H_{\mathrm{I}}(t') &= \frac{(2\pi)^3\delta(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3)}{a^{-3}\zeta^2} \int \prod_{j=1}^{3} \frac{d^3q_j}{(2\pi)^3} \sum_{s_j} h^{s_j}_{\mathbf{q}_j}(t')\\
&\times \Bigg[\left(\bar{A}-1\right) R + \frac{g_3}{\zeta^2}\left(R^{ij}R_{ij}\right)\\
&\quad + \frac{g_8}{\zeta^4}\left(\nabla^i R^{jk}\right)\left(\nabla_i R_{jk}\right) + \frac{g_6}{\zeta^4} R^i_j R^j_k R^k_i \Bigg],
\end{aligned} \tag{3.18}$$

where expressions of R, $\left(R^{ij}R_{ij}\right)$, $(\nabla^i R^{jk})(\nabla_i R_{jk})$ and $R^i_j R^j_k R^k_i$ are given in Eqs.(3.12-3.15).

Promoting the scalar variable $h^s_{\mathbf{k}}(t)$ to a quantized field

$$\hat{h}^s_{\mathbf{k}}(t) = h_{\mathbf{k}}(t)\hat{a}_s(\mathbf{k}) + h^*_{-\mathbf{k}}(t)\hat{a}^\dagger_s(-\mathbf{k}), \tag{3.19}$$

where the creation and annihilation operators satisfy the commutation relation

$$\left[\hat{a}_s(\mathbf{k}), \hat{a}^\dagger_{s'}(\mathbf{k}')\right] = (2\pi)^3\,\delta(\mathbf{k}-\mathbf{k}')\delta_{ss'}, \tag{3.20}$$

we are now in the position to employ the in-in formalism [23]. The 3-point correlator we seek can be calculated at leading order

$$\begin{aligned}
&\langle \hat{h}^{s_1}_{\mathbf{k}_1}(t)\,\hat{h}^{s_2}_{\mathbf{k}_2}(t)\,\hat{h}^{s_3}_{\mathbf{k}_3}(t)\rangle\\
&\simeq i\int_{t_i}^{t} dt' \langle \left[\hat{H}_{\mathrm{I}}(t'), \hat{h}^{s_1}_{\mathbf{k}_1}(t)\,\hat{h}^{s_2}_{\mathbf{k}_2}(t)\,\hat{h}^{s_3}_{\mathbf{k}_3}(t)\right]\rangle\\
&= i\,(2\pi)^3\,\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \int_{t_i}^{t} dt' \frac{a^3(t')}{\zeta^2}\\
&\quad\times \Big\{ F_{s_1s_2s_3}(-\mathbf{K},t') \left[W(\mathbf{K},t';t) - W^*\right] \Big\}\\
&\qquad + \ 5 \text{ permutations of } (\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3),
\end{aligned} \tag{3.21}$$

where we have defined the product of mode functions

$$W(\mathbf{K},t';t) \equiv h_{k_1}(t')h^*_{k_1}(t)h_{k_2}(t')h^*_{k_2}(t)h_{k_3}(t')h^*_{k_3}(t), \tag{3.22}$$

and for the contraction $\{(\mathbf{q}_1,\mathbf{k}_1),(\mathbf{q}_2,\mathbf{k}_2),(\mathbf{q}_3,\mathbf{k}_3)\}$

$$\begin{aligned}
&F_{s_1s_2s_3}(-\mathbf{K},t')\\
&\equiv \left(\frac{\bar{A}-1}{a^2(t')}\right)\frac{1}{6}\Big[C_1 - \frac{1}{4}C_2 - \frac{1}{4}C_3\Big]^*_{\mathbf{k}}(123)\\
&\quad - \frac{g_3}{a^4(t')\zeta^2}\frac{k_1^2+k_2^2+k_3^2}{6}\Big[C_1 - \frac{1}{4}C_2 - \frac{1}{4}C_3\Big]^*_{\mathbf{k}}(123)\\
&\quad - \frac{g_8}{a^6(t')\zeta^4}\frac{k_1^4+k_2^4+k_3^4}{6}\Big[C_1 - \frac{1}{4}C_2 - \frac{1}{4}C_3\Big]^*_{\mathbf{k}}(123)\\
&\quad - \frac{g_8}{a^6(t')\zeta^4}\frac{k_1^2k_3^2}{6}\Big[C_1 - \frac{1}{4}C_2\Big]^*_{\mathbf{k}}(123)\\
&\quad - \frac{g_8}{a^6(t')\zeta^4}\frac{k_1^2k_2^2}{6}\Big[C_1 - \frac{1}{4}C_3\Big]^*_{\mathbf{k}}(123)\\
&\quad + \frac{g_6}{a^6(t')\zeta^4}\frac{k_1^2k_2^2k_3^2}{24}\Big[C_5\Big]^*_{\mathbf{k}}(123) + \text{cyclic},
\end{aligned} \tag{3.23}$$

with

$$\begin{aligned}
\Big[C_1\Big]^*_{\mathbf{k}}(123) &\equiv k_{1a}k_{1b}\varepsilon^{s_1}_{ij}(-\mathbf{k}_1)\,\varepsilon^{s_2}_{ia}(-\mathbf{k}_2)\,\varepsilon^{s_3}_{jb}(-\mathbf{k}_3)\\
&= k_{1a}k_{1b}\varepsilon^{-s_1}_{ij}(\mathbf{k}_1)\,\varepsilon^{-s_2}_{ia}(\mathbf{k}_2)\,\varepsilon^{-s_3}_{jb}(\mathbf{k}_3)\\
&= k_{1a}k_{1b}\Big[\varepsilon^{s_1}_{ij}(\mathbf{k}_1)\,\varepsilon^{s_2}_{ia}(\mathbf{k}_2)\,\varepsilon^{s_3}_{jb}(\mathbf{k}_3)\Big]^*,
\end{aligned} \tag{3.24}$$

and similarly for $\Big[C_2\Big]^*_{\mathbf{k}}(123)$ and $\Big[C_3\Big]^*_{\mathbf{k}}(123)$. We see that the first line of (3.23) is contributed by R, the second line by $R^{ij}R_{ij}$, the third, fourth and fifth lines by $\left(\nabla^i R^{jk}\right)(\nabla_i R_{jk})$, and the last line by $R^i_j R^j_k R^k_i$.

IV. THE MODE INTEGRATION

As was noted in [19], the k-dependence of the bispectrum receives contributions from both the interaction Hamiltonian and the mode function, whose behavior is determined by the EoM and its initial state (mathematically the initial condition). In principle, one should perform the integration in (3.21) tracing the behavior of the mode function through out its history from the initial time (t_i) until several e-folds after horizon exit when the mode freezes (t). However, during the early times the modes are highly oscillatory and the integration during that time should be mostly canceled out by itself. This, of course, requires some conditions on the integrands.

Looking at the integrands in (3.21), we see that the non-oscillating time-dependent parts come from the common factor $a^3(t')$, the factor of $a^{-2n}(t')$ in $F_{s_1s_2s_3}$ ($n = 1, 2, 3$) and from the three mode functions. The oscillating time-dependent part in the mode function is modeled as [19]

$$h \propto (\eta')^m \exp[i\lambda_l k^l (\eta')^l], \tag{4.1}$$

where m and l are dynamical, i.e. they change when different terms dominate the dispersion relation. Since

we are working with de Sitter space, then $a = -1/(H\eta)$, hence the integration can be written in a schematic form

$$\int d\eta' \exp[i\lambda_l \left(k_1^l + k_2^l + k_3^l\right) \eta'^l] \eta'^{3m+(2n-4)}. \qquad (4.2)$$

When $3m + (2n-4) \le 0$, the integration must be small because $\eta'^{3m+(2n-4)}$ changes very slowly during the early times ($\eta' \to -\infty$) while the oscillating part has high frequencies; similarly when $0 < 3m+(2n-4) < l$, the oscillation smoothes out the changes in $\eta'^{3m+(2n-4)}$ as one can always perform a change of variables $\tau = \eta'^{3m+(2n-4)+1}$ and integrate over τ with a purely oscillating integrand. When $3m + (2n-4) \ge l$, on the other hand, care is needed. However, if $3m + (2n-4)$ is not so higher than l, the errors introduced by ignoring this history should not be too large. In fact for the current model, when k^6 term dominates the dispersion relation, $m = 0, l = 3$ and $n_{\text{MAX}} = 3$, hence $3m + (2n-4) \le 2 < l$. Therefore we shall work under this assumption below, and consider only the period when the k^2 term dominates the dispersion. During this period, the oscillating mode function takes the form,

$$\begin{aligned} v_k(t) &= \frac{C_+(k\eta_i)}{\sqrt{2k}} \left(1 - \frac{1}{ic_T k\eta}\right) e^{-ic_T k\eta} \\ &\quad + \frac{C_-(k\eta_i)}{\sqrt{2k}} \left(1 + \frac{1}{ic_T k\eta}\right) e^{ic_T k\eta}, \qquad (4.3) \end{aligned}$$

where $c_T^2 \equiv (1-\bar{A})$, the canonically normalized field $v_k(t)$ is related to $h_k(t)$ through[2]

$$v_k(t) = \frac{aM_{\text{pl}}}{2} h_k(t), \qquad (4.4)$$

and the constants C_+ and C_- are in general functions of H, M_{pl}, the new mass scale M_* and transition time η_i [19]. The reason for this dependence on $(k\eta_i)$ is the UV stage when the dispersion relation differs from the relativistic form significantly. The detailed form however, depends on the procedure of matching of this "relativistic solution" with the solution in the UV region. One example is the matching we considered in [19].

Summing up the discussions, we see that the UV history has at least three effects on the integration we are considering: one is that η_i can no longer be extended to Euclidean space $-\infty(1+i\epsilon)$; second, the dependence of the normalization on η_i; third, if the mode underwent a non-adiabatic period ($\omega^2 < 2/\eta^2$), then a "negative frequency" branch exists ($C_- \neq 0$). Since we've seen in [19] that a mixture of "negative frequency" and "positive frequency" modes gives an enhanced folded shape in the bispectrum [19], in this paper we focus on the case when $C_- = 0$.

[2] Note that the relation here is different from the relation in [18] due to the different normalization we choose for the polarization tensors. The EoM's however, are the same for both.

The bispectrum is then given as

$$\begin{aligned} &\langle \hat{h}^{s_1}_{\mathbf{k}_1}(t)\, \hat{h}^{s_2}_{\mathbf{k}_2}(t)\, \hat{h}^{s_3}_{\mathbf{k}_3}(t) \rangle \\ &= (2\pi)^3 \delta\left(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3\right) \zeta^{-2} \left| \frac{\sqrt{2} H C_+}{c_T M_{\text{pl}}} \right|^6 \frac{2G^*_{s_1 s_2 s_3}(\mathbf{K})}{k_1^3 k_2^3 k_3^3} \\ &\quad + \mathcal{E}(\text{UV}) + \mathcal{E}(\text{Finite } \eta_i) + 5 \text{ perm.'s}, \qquad (4.5) \end{aligned}$$

where we have introduced two error terms signifying that we considered only the relativistic region in the integration, and defined, for the contraction pairing $\{(\mathbf{q}_1, \mathbf{k}_1), (\mathbf{q}_2, \mathbf{k}_2), (\mathbf{q}_3, \mathbf{k}_3)\}$,

$$\begin{aligned} &G^*_{s_1 s_2 s_3}(\mathbf{K}) \\ &\equiv \left(\bar{A} - 1\right) \mathcal{I} \frac{1}{6} \Big[C_1 - \frac{1}{4} C_2 - \frac{1}{4} C_3\Big]^*_{\mathbf{k}} (123) \\ &\quad - \frac{g_3}{\zeta^2} \mathcal{II} \frac{k_1^2 + k_2^2 + k_3^2}{6} \Big[C_1 - \frac{1}{4} C_2 - \frac{1}{4} C_3\Big]^*_{\mathbf{k}} (123) \\ &\quad - \frac{g_8}{\zeta^4} \mathcal{III} \frac{k_1^4 + k_2^4 + k_3^4}{6} \Big[C_1 - \frac{1}{4} C_2 - \frac{1}{4} C_3\Big]^*_{\mathbf{k}} (123) \\ &\quad - \frac{g_8}{\zeta^4} \mathcal{III} \frac{k_1^2 k_3^2}{6} \Big[C_1 - \frac{1}{4} C_2\Big]^*_{\mathbf{k}} (123) \\ &\quad - \frac{g_8}{\zeta^4} \mathcal{III} \frac{k_1^2 k_2^2}{6} \Big[C_1 - \frac{1}{4} C_3\Big]^*_{\mathbf{k}} (123) \\ &\quad + \frac{g_6}{\zeta^4} \mathcal{III} \frac{k_1^2 k_2^2 k_3^2}{24} \Big[C_5\Big]^*_{\mathbf{k}} (123) + \text{cyclic}, \qquad (4.6) \end{aligned}$$

where

$$\begin{aligned} \mathcal{I} &= \frac{-\left(k_1 + k_2 + k_3\right)^3 + k_1 k_2 k_3}{c_T^{-1} \left(k_1 + k_2 + k_3\right)^2} \\ &\quad + \frac{\left(k_1 + k_2 + k_3\right)\left(k_1 k_2 + k_2 k_3 + k_3 k_1\right)}{c_T^{-1} \left(k_1 + k_2 + k_3\right)^2}, \\ \mathcal{II} &= -2 \frac{\left(k_1 + k_2 + k_3\right)^3 + 3 k_1 k_2 k_3}{c_T \left(k_1 + k_2 + k_3\right)^4} \\ &\quad -2 \frac{\left(k_1 + k_2 + k_3\right)\left(k_1 k_2 + k_2 k_3 + k_3 k_1\right)}{c_T \left(k_1 + k_2 + k_3\right)^4}, \\ \mathcal{III} &= 8 \frac{\left(k_1 + k_2 + k_3\right)^3 + 15 k_1 k_2 k_3}{c_T^3 \left(k_1 + k_2 + k_3\right)^6}, \\ &\quad + 24 \frac{\left(k_1 + k_2 + k_3\right)\left(k_1 k_2 + k_2 k_3 + k_3 k_1\right)}{c_T^3 \left(k_1 + k_2 + k_3\right)^6}. \qquad (4.7) \end{aligned}$$

We see that the magnitude of the bispectrum depends on C_+ which in turn depends on the new energy scale M_*. Since we've found in [18] that the general relativistic value of the power spectrum for tensor perturbations is obtainable in the HMT framework when $H \ll M_*$, we can have the same conclusion as that for scalar perturbations presented in [19]: a large bispectrum is possible, provided that M_* is not too much lower than the Planck scale.

V. SHAPE OF THE BISPECTRUM

We are now ready to plot the shapes of the bispectrum. For $s_1 = s_2 = s_3 = 1$ and $s_1 = s_2 = -s_3 = 1$, we plot the shapes contributed by various terms separately in Fig.'s 1 and 2. These two figures represent all possible configurations when we do not have parity violating terms.

When spins of all 3 tensor fields are the same ($s_1 = s_2 = s_3 = 1$), the signal generated by the relativistic term R peaks at the squeezed limit ($k_3/k_1 \to 0$), while the term $R^i_j R^j_k R^k_i$ favors the equilateral shape ($k_2/k_1 \simeq k_3/k_1 \simeq 1$). It's interesting to note that the other two terms, $R^{ij}R_{ij}$ and $\left(\nabla^i R^{jk}\right)\left(\nabla_i R_{jk}\right)$, also generate larger signal in the squeezed limit, though they are of higher-order derivatives. This is indeed expected, if one realizes that the k-dependence of them are similar to that of the GR term [cf. Eqs.(3.12-3.15)].

When the spins are mixed ($s_1 = s_2 = -s_3 = 1$), the terms $R, R^{ij}R_{ij}$ and $\left(\nabla^i R^{jk}\right)\left(\nabla_i R_{jk}\right)$ generate shapes similar to the previous case. A particular interesting result is that the signal generated by $R^i_j R^j_k R^k_i$ no longer favors the equilateral shape but peaks in between the equilateral and squeezed limits. This can be understood as that for the mixed spin the product of the polarization tensors gives a strong favor of the squeezed shape. To illustrate this, we plot the k-dependence of them in both cases $(+++)$ and $(++-)$ in Figs. 3 and 4, where we've defined

$$
\begin{aligned}
\text{Configuration1} &= \left[C_1 - \frac{1}{4}C_2 - \frac{1}{4}C_3\right]^*_{\mathbf{k}}(123) + \text{ cyclic},\\
\text{Configuration2} &= k_1^2 k_3^2\left[C_1 - \frac{1}{4}C_2\right]^*_{\mathbf{k}}(123) + \text{ cyclic},\\
\text{Configuration3} &= k_1^2 k_2^2\left[C_1 - \frac{1}{4}C_3\right]^*_{\mathbf{k}}(123) + \text{ cyclic},\\
\text{Configuration4} &= \left[C_5\right]^*_{\mathbf{k}}(123) + \text{ cyclic}. \qquad (5.1)
\end{aligned}
$$

VI. CONSTRAINTS FROM PLANCK OBSERVATIONS

Finally, we would like to comment on the recently-released results of Planck on primordial non-Gaussianity [24]. As noted at the end of Section IV, the magnitude of the bispectrum is dependent on the mass scale M_*. Here we use the Planck result to obtain a constraint on M_*.

Very roughly speaking, the non-linearity parameter for bispectrum can be estimated as

$$f^{\mathrm{T}}_{\mathrm{NL}} \sim \frac{\langle hhh\rangle}{\langle hh\rangle\langle hh\rangle} = \frac{\langle hhh\rangle}{\left[\Delta^2_{\mathrm{T}}\right]^2}, \qquad (5.1)$$

where Δ^2_T is the scale-invariant power spectrum of the tensor perturbations. It is natural to assume that the

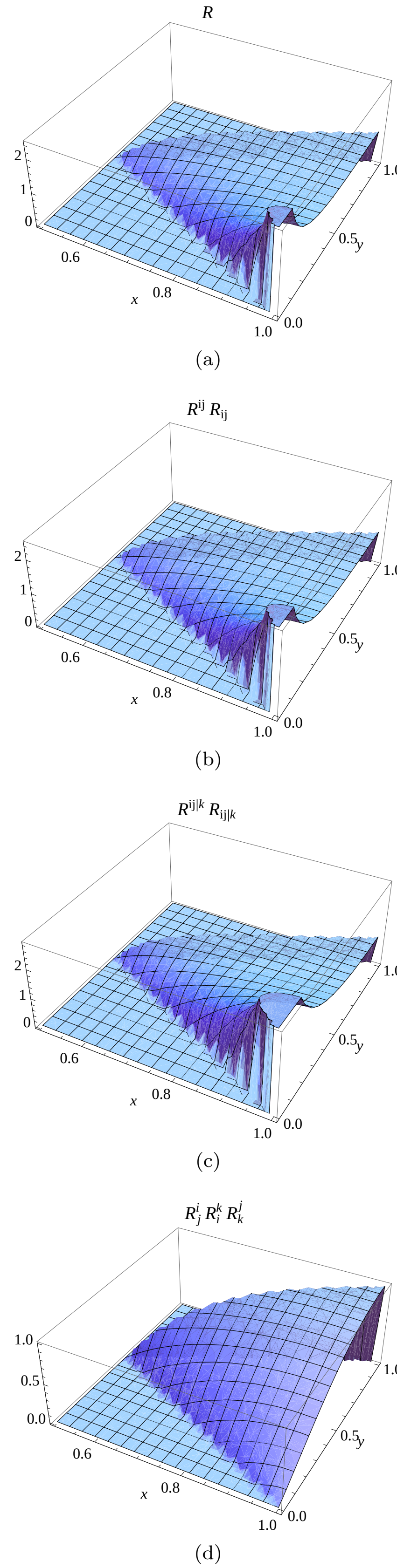
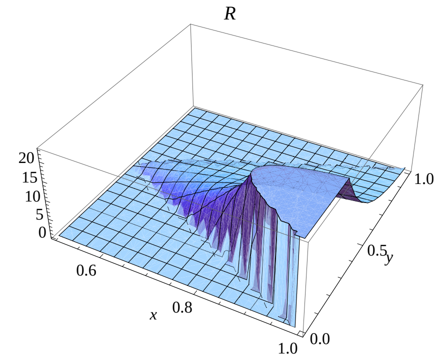
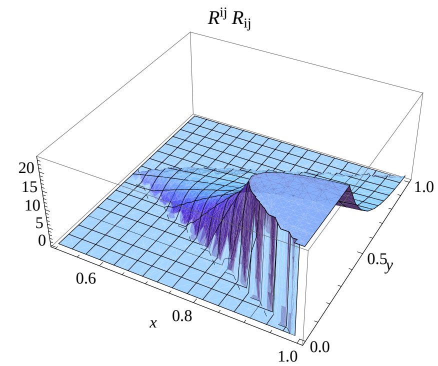


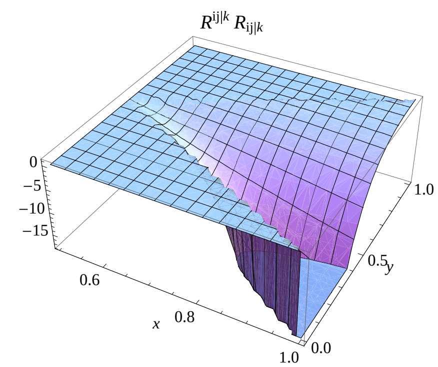
FIG. 1: Shapes of $(k_1k_2k_3)^{-1}G_{+++}(\mathbf{K})$ contributed by various terms. All are normalized to unity for equilateral limit. $x = k_2/k_1, y = k_3/k_1$.



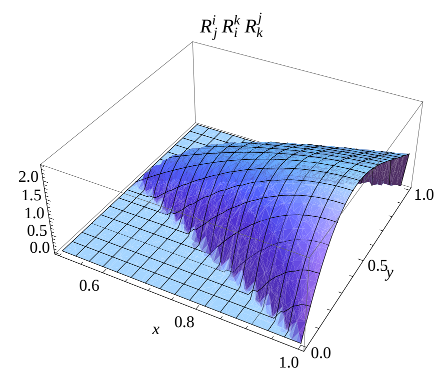
(a)



(b)

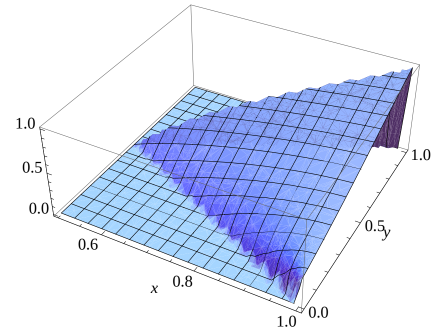


(c)

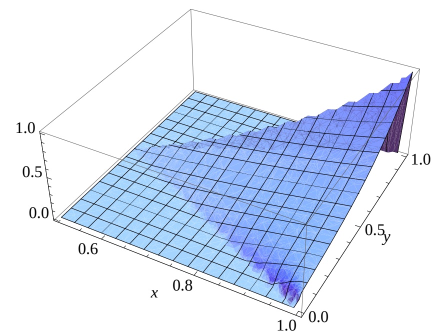


(d)

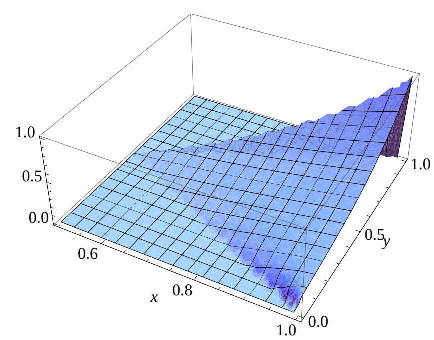
FIG. 2: Shapes of $(k_1k_2k_3)^{-1}G_{++-}(\mathbf{K})$ contributed by various terms. All are normalized to unity for equilateral limit. $x = k_2/k_1, y = k_3/k_1$.



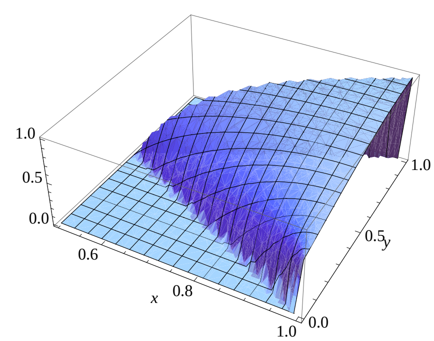
(a)Configuration 1



(b)Configuration 2

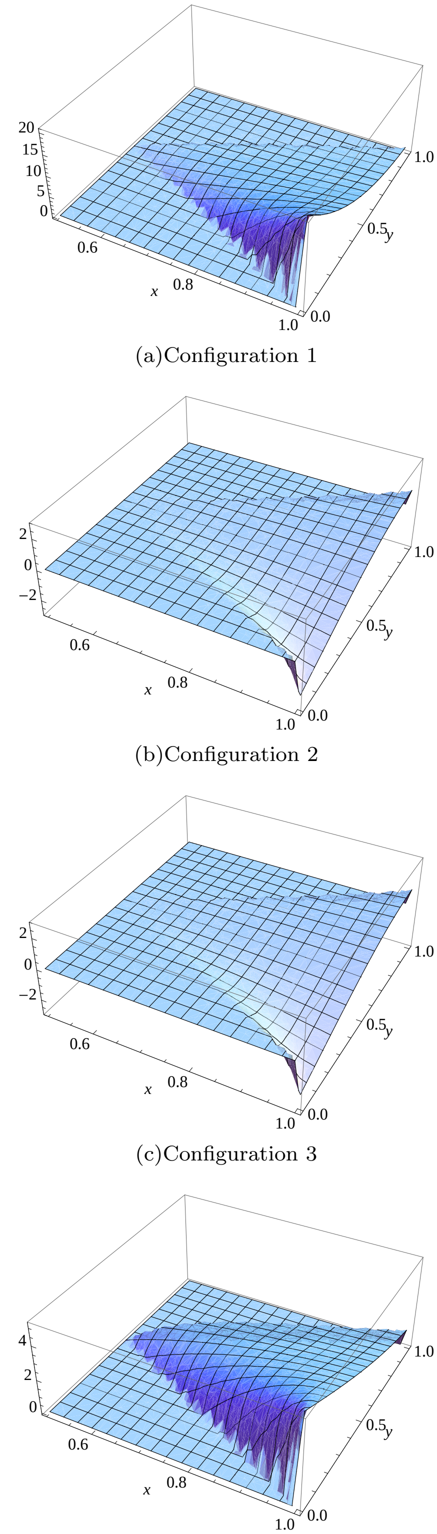


(c)Configuration 3



(d)Configuration 4

FIG. 3: Shapes of the different configurations of the polarization tensors for $s_1 = s_2 = s_3 = 1$. All are normalized to unity for equilateral limit. $x = k_2/k_1, y = k_3/k_1$.



(a)Configuration 1

(b)Configuration 2

(c)Configuration 3

(d)Configuration 4

FIG. 4: Shapes of the different configurations of the polarization tensors for $s_1 = s_2 = -s_3 = 1$. All are normalized to unity for equilateral limit. $x = k_2/k_1, y = k_3/k_1$.

bispectrum of the tensor perturbations has a lower magnitude than does the scalar perturbations,

$$\langle hhh \rangle \lesssim \langle \mathcal{RRR} \rangle. \tag{5.2}$$

Hence

$$\begin{aligned} \frac{\langle hhh \rangle}{\left[\Delta_{\mathrm{T}}^2\right]^2} &\lesssim r^{-2} \frac{\langle \mathcal{RRR} \rangle}{\left[\Delta_{\mathcal{R}}^2\right]^2} \sim r^{-2} f_{\mathrm{NL}}^{\mathcal{R}}, \\ \langle hhh \rangle &\lesssim f_{\mathrm{NL}}^{\mathcal{R}} \left[\Delta_{\mathcal{R}}^2\right]^2, \end{aligned} \tag{5.3}$$

where r is the tensor-to-scalar ratio and $\Delta_{\mathcal{R}}^2$ is the scale-invariant power spectrum of the scalar perturbations which takes the value $\left(4.9 \times 10^{-5}\right)^2$.

The equilateral signal in our model takes the approximate value

$$\langle hhh \rangle \sim \left| \frac{HC_+}{c_{\mathrm{T}}} \right|^6 g_6, \tag{5.4}$$

where we've taken $M_{\mathrm{pl}} \equiv 1$. The non-linearity parameter for the equilateral shape is constrained by the Planck result $f_{\mathrm{NL}}^{\mathcal{R}} = -42 \pm 75$, or $|f_{\mathrm{NL}}^{\mathcal{R}}| \lesssim 100$. This, together with Eqs.(5.3) and (5.4), requires

$$\left| \frac{HC_+}{c_{\mathrm{T}}} \right|^6 |g_6| \lesssim 100 \left(4.9 \times 10^{-5}\right)^4 \lesssim \left(10^{-4}\right)^4. \tag{5.5}$$

The normalization condition for the mode function v_k in the relativistic region in (4.3) reads

$$v_k v_k'^* - v_k^* v_k' = i\hbar, \tag{5.6}$$

which requires

$$\frac{C_+}{\sqrt{c_T}} \simeq 1. \tag{5.7}$$

In the mean time, good IR behavior (so that the theory flows to GR in this limit) requires $c_T \simeq 1$, hence $C_+ \simeq 1$. This implies

$$\frac{C_+}{c_T} \simeq 1 \simeq \frac{C_+}{\sqrt{c_T}}. \tag{5.8}$$

Apply this to condition (5.5), we have

$$H^6 |g_6| \lesssim (10^{-4})^4. \tag{5.9}$$

Since g_6 and g_8 are both coupling constants for the cubic curvature terms, it's natural to assume that

$$|g_6| \simeq |g_8|. \tag{5.10}$$

This gives us the final constraint

$$\frac{M_*}{M_{\mathrm{pl}}} \epsilon_{HL}^{3/2} = \frac{M_*}{M_{\mathrm{pl}}} \left(\frac{H}{M_*}\right)^3 \lesssim 10^{-8}. \tag{5.11}$$

where we've written M_{pl} explicitly and $\epsilon_{HL} \equiv H^2/M_*^2$ was introduced in [18].

In the nonprojectable case without the extra U(1) symmetry, the frame effects imposed the most stringent constraint on the upper bound, $M_* \lesssim 10^{16}$ GeV [9], while in the case with the local U(1) symmetry, such effects have not been worked out yet, either with the projectable condition or without it. Assuming that this is also true in the present case, the above condition implies that $H/M_* \lesssim 10^{-2}$. For $M_* \simeq M_{pl}$, we find that $H/M_* \lesssim 10^{-3}$.

On the other hand, in [18] it was found that when $\epsilon_{HL} = (H/M_*)^2 \ll 1$, our model can reproduce the power spectrum given by the inflationary models in GR. Taking $\epsilon_{HL} \sim \mathcal{O}(10^{-2})$, we find that constraint (5.11) gives $M_*/M_{\rm pl} \lesssim 10^{-5}$.

VII. CONCLUSIONS AND REMARKS

In this paper, we study 3-point correlation function of primordial gravitational waves generated during the de Sitter expansion of the universe in the framework of the general covariant Hořava-Lifshitz gravity with the projectability condition and an arbitrary coupling constant λ. We find that the interaction Hamiltonian, at leading order, receives contribution from four terms built of the 3-dimensional curvature:

$$R, \quad R_{ij}R^{ij}, \quad R^i_jR^j_kR^k_i, \quad \left(\nabla^i R^{jk}\right)\left(\nabla_i R_{jk}\right).$$

The Ricci scalar R yields the same k-dependence as that in general relativity, i.e. its signal peaks at the squeezed limit regardless of the spins of the tensor fields, but with different magnitude due to coupling with the $U(1)$ field A and a UV history when the dispersion relation is significantly different from the relativistic form. The two terms $R_{ij}R^{ij}$ and $\left(\nabla^i R^{jk}\right)\left(\nabla_i R_{jk}\right)$ generate shapes similarly to the R term. The term $R^i_jR^j_kR^k_i$ favors the equilateral shape when spins of the three tensor fields are the same but peaks in between the equilateral and squeezed limits when spins are mixed. We find that this is due to the effect of the polarization tensors: when spins are mixed, the product of the three polarization tensors strongly favors the squeezed shape. Thus, an absence of the equilateral shape in the bispectrum of the tensor perturbations cannot rule out gravity theories of higher order derivatives, at least for the R^2 type of theories.

It should be noted that in performing the integration over the three mode functions to get the full expression of the bispectrum [cf. Eq.(4.5)], we integrated over only the region when k^2 dominated the dispersion relation [cf. Eq.(4.3)]. We gave a qualitative argument for the condition under which the UV history can be (partly) ignored, in leading order analysis. A quantitative study of the errors introduced with such ignorance will certainly deserve further analysis.

Finally, we've also obtained a constraint on M_* and H [cf. Eq.(5.11)] using the Planck results, released recently [24].

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